

## Area Of Study 1, Motion, Study Notes 2

### Distance and Displacement

**Distance** is a **scalar** quantity of how far an object has travelled, measured in **metres**. Distance takes into account the entire path of the object, including bends and turns.

**Displacement** is a **vector** quantity, of the distance **and** direction, in a **straight line** between an object's starting and finishing points. Like distance, the magnitude of displacement is measured in metres.

In 1 dimensional motion (movement on only either a horizontal or vertical plane), directions of displacement (and velocity and acceleration) are usually indicated by a **positive magnitude for forward or up**, or by a **negative magnitude for backward or down**.

### Speed and Velocity

**Speed** is a scalar quantity, most simply defined as how "fast" an object moves, without regard for the direction in which it moves. Speed can be thought of more accurately as how quickly distance changes as time passes, or mathematically, as the **rate** at which distance changes, or as of a change in distance in an amount of time.

$$speed = \frac{distance}{time}$$

**Velocity** is similar to speed, but it relates **displacement** and time, **not** distance and time, so it's a **vector** quantity. Velocity is the rate at which displacement (in terms of both magnitude **and** direction) changes, or a change in displacement in an amount of time.

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{\bar{s}_2 - \bar{s}_1}{t_2 - t_1}$$

Where  $\Delta s$  = change in displacement, in m

$\Delta t$  = change in time, in s

$s_1$  = initial displacement, in m

$s_2$  = final displacement, in m

$t_1$  = initial time, in s

$t_2$  = final time, in s

$v$  = velocity, in  $\text{ms}^{-1}$

**Danger ⚡**: Don't confuse the pronumeral  $s$  for displacement, with  $s$  for seconds, the unit of time.

**Average velocity** is the **total displacement** divided by the **total time** for a moving object, giving a **single velocity** for the **entire duration** of its motion.

**Instantaneous velocity** is the velocity of an object at any single distinct moment in time. This is mathematically impossible, as  $\Delta t = 0$ , making  $v$  undefined. Instantaneous velocity is actually a change in displacement over a **very small** change in time.

## Acceleration

**Acceleration** is a vector quantity, and can be thought of as how quickly velocity changes as time passes, or mathematically, as the rate at which velocity (in terms of both magnitude **and** direction) changes, or a change in velocity in an amount of time.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{\bar{v}_2 - \bar{v}_1}{t_2 - t_1} = \frac{\bar{v} - \bar{u}}{t_2 - t_1}$$

Where  $\Delta v$  = change in velocity, in  $\text{ms}^{-1}$   
 $\Delta t$  = change in time, in s  
 $v_1 = u$  = initial velocity, in  $\text{ms}^{-1}$   
 $v_2 = v$  = final velocity, in  $\text{ms}^{-1}$   
 $t_1$  = initial time, in s  
 $t_2$  = final time, in s  
 $a$  = acceleration, in  $\text{ms}^{-2}$

## Graph Interpretation

Displacement ( $s$ ), velocity ( $v$ ), and acceleration ( $a$ ) can each be graphed against time ( $t$ ). Time is always on the horizontal ( $x$ ) axis, with the other variable ( $s$ ,  $v$  or  $a$ ) on the vertical ( $y$ ) axis.

Because  $s$ ,  $v$  and  $a$  are all related, a graph of any one of these variables against time can give information about the other variables:

- The gradient of an object's  $s/t$  graph gives the object's  $v$ , for any particular value of  $t$ .
- The gradient of an object's  $v/t$  graph gives the object's  $a$ , for any particular value of  $t$ .
- The area bound by an object's  $v/t$  graph gives the object's  $\Delta s$ , for any particular  $\Delta t$ .
- The area bound by an object's  $a/t$  graph gives the object's  $\Delta v$ , for any particular  $\Delta t$ .

Note that:

For an  $s/t$  graph,  $\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{\Delta \bar{s}}{\Delta t} = \bar{v}$ . The formulas for gradient and velocity are identical.

Likewise:

For an  $v/t$  graph,  $\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{\Delta \bar{v}}{\Delta t} = \bar{a}$ . The formulas for gradient and acceleration are identical.

Of course, if a  $s/t$  or  $v/t$  graph is curved, it is the gradient of the **tangent** to the curve that gives the  $v$  or  $a$  for any particular value of  $t$ .

Methods for finding the area bound by a graph within limits of time may be simple geometry, "square counting" or calculus.

## Constant Acceleration Formulæ

As long as the acceleration of an object is constant or “uniform” (doesn’t change) the below formulæ relate all of the involved variables to allow for the solution of unknown magnitudes.

The derivation of these formulæ is from the combination and simplification of the mathematical procedures used to interpret the graphs described in the previous section.

Of course re-arrangement is often necessary for finding an unknown value. Also, an exam problem often requires more than one step to reach a final solution. For example, application of one formula may be required in order to gain the value of a variable needed to apply another formula so as to find a problem’s required solution.

$$\bar{s} = \bar{u}t + \frac{1}{2}\bar{a}t^2$$

$$v^2 = u^2 + 2\bar{a}\bar{s}$$

Where  $s$  = displacement, in m  
 $u$  = initial velocity, in  $\text{ms}^{-1}$   
 $v$  = final velocity, in  $\text{ms}^{-1}$   
 $t$  = time, in s  
 $a$  = acceleration, in  $\text{ms}^{-2}$

**Danger ⚡:** Again don’t confuse the pronumerals  $s$  for displacement, with  $s$  for seconds, the unit of time.