# Area Of Study 1, Motion In One And Two Dimensions, Study Notes 2

## Distance and Displacement

**Distance** is a **scalar** quantity of how far an object has travelled, measured in **metres**. Distance takes into account the entire path of the object, including bends and turns.

**Displacement** is a **vector** quantity, of the distance **and** direction, in a **straight line** between an object's starting and finishing points. Like distance, the magnitude of displacement is measured in metres.

In 1 dimensional motion (movement on only either a horizontal or vertical plane), directions of displacement (and velocity and acceleration) are usually indicated by a **positive magnitude for forward or up**, or by a **negative magnitude for backward or down**.

## Speed and Velocity

**Speed** is a scalar quantity, most simply defined as how "fast" an object moves, without regard for the direction in which it moves. Speed can be thought of more accurately as how quickly distance changes as time passes, or mathematically, as the **rate** at which distance changes, or as of a change in distance in an amount of time.

$$speed = \frac{distance}{time}$$

**Velocity** is similar to speed, but it relates **displacement** and time, **not** distance and time, so it's a **vector** quantity. Velocity is the rate at which displacement (in terms of both magnitude **and** direction) changes, or a change in displacement in an amount of time.

$$\vec{v} = \frac{\Delta s}{\Delta t} = \frac{\vec{s}_2 - \vec{s}_1}{t_2 - t_1}$$

Where $\Delta s$ = change in displacement, in m
$\Delta t$ = change in time, in s
$s_1$ = initial displacement, in m
$s_2$ = final displacement, in m
$t_1$ = initial time, in s
$t_2$ = final time, in s
$v$ = velocity, in $ms^{-1}$

**Danger** ☠: Don't confuse the pronumeral $s$ for displacement, with **s** for seconds, the unit of time.

**Average velocity** is the **total displacement** divided by the **total time** for a moving object, giving a **single velocity** for the **entire duration** of its motion.

**Instantaneous velocity** is the velocity of an object at any single distinct moment in time. This is mathematically impossible, as $\Delta t$ = 0, making $v$ undefined. Instantaneous velocity is actually a change in displacement over a **very small** change in time.

## Frames of Reference

A **frame of reference** is a perspective from a specific location of an object's motion. The location from where an object's motion is observed is important, because it determines the direction from which the motion is seen. The direction from where an object's motion is observed determines the direction of the object's motion **relative to the observer**.

If an object's motion is observed from the perspective, or "frame of reference", of a moving location, the magnitudes involved with the object's motion are also affected (again, relative to the observer).

Considering a person in a shopping trolley cruising in a straight line:

- For an observer on **one side** of the trolley, it's moving from **left to right**.
- For an observer on **the other side** of the trolley, it's moving from **right to left** (the opposite direction).
- For an observer **behind** the trolley, it's moving **away from them** (in a **positive**, or **forwards** direction).
- For an observer **in front of** the trolley, it's moving **towards them** (in a **negative**, or **backwards** direction).
- For the observer **in** the trolley, it's **stationary** (as the trolley is moving **with** them).

Examples in which frames of reference have an obvious effect on observations of motion:

- A small car stationary between two large trucks, all facing the same direction. The two large trucks begin accelerating **forwards**. An observer in the stationary small car senses the same magnitude of acceleration, but in a **backwards** direction. From either of the two trucks' frame of reference, the small car **is** accelerating backwards.
- Two trains travelling at the **same velocity**, side by side. From the frame of reference of an observer on either train, the other train is **stationary**.
- Continuing the above example, but if one train **accelerates** slightly. From the frame of reference of an observer on the accelerating train, the other train is **accelerating backwards** (or decelerating).
- A more complex situation: A ball dropped by someone standing on a train station platform, while a train accelerates away. From the frame of reference of the "ball-dropper", the ball accelerates only **downwards**. From the frame of reference of a passenger on the train, the ball accelerates downwards **and backwards** simultaneously. From the frame of reference of a passenger **facing backwards** on another train and travelling with **constant velocity** in the **opposite direction**, the ball is travelling with a **constant forwards velocity** while it's accelerating downwards.

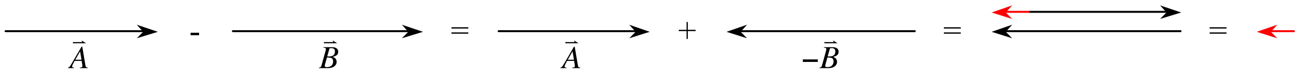
## Relative Velocity

**Relative velocity** is the velocity of one object in relation to another. For the analysis of motion previous to the consideration of frames of reference, velocity has been regarded from the perspective of a stationary observer in a fixed location. When an object's motion can be observed from the frame of reference of another moving object, each of the two objects has a velocity in relation to the other, which is different from either of their velocities from the frame of reference of a stationary observer in a fixed location. The relative velocity of one object to another is usually found by vector subtraction. To know which vector to subtract from which (in order to get the direction correct), it helps to visualise yourself as one of the objects observing the other.

Example:

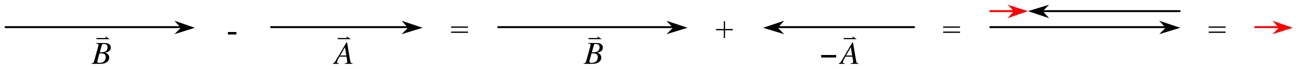
From the frame of reference of an observer in a fixed location:
Car A has a velocity of 20ms$^{-1}$ (or 72km/h), so let $\vec{A} = 20ms^{-1}$.
Car B has a velocity of 25ms$^{-1}$ (or 90km/h), so let $\vec{B} = 25ms^{-1}$.
Note that both cars are travelling in the same direction.

To find car A's velocity relative to car B, use $\vec{A} - \vec{B}$.




So, relative to car B, car A has a velocity of -5ms$^{-1}$ (18km/h **backwards**).

To find Car B's velocity relative to Car A, use $\vec{B} - \vec{A}$.




So, relative to car A, car B has a velocity of 5ms$^{-1}$ (18km/h **forwards**).

Other examples could involve cars A and B with the same magnitude of velocity, but:

- Travelling in opposite directions (approaching or going away from each other).
- Travelling in directions at any angle to each other (diverging or converging directions).

The process of vector subtraction could be used to find the relative velocities in each of the above situations.

## Acceleration

**Acceleration** is a vector quantity, and can be thought of as how quickly velocity changes as time passes, or mathematically, as the rate at which velocity (in terms of both magnitude **and** direction) changes, or a change in velocity in an amount of time.

$$\vec{a} = \frac{\Delta v}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\vec{v} - \vec{u}}{t_2 - t_1}$$

Where $\Delta v$ = change in velocity, in ms$^{-1}$
$\Delta t$ = change in time, in s
$v_1$ = $u$ = initial velocity, in ms$^{-1}$
$v_2$ = $v$ = final velocity, in ms$^{-1}$
$t_1$ = initial time, in s
$t_2$ = final time, in s
$a$ = acceleration, in ms$^{-2}$

## Graph Interpretation

Displacement ($s$), velocity ($v$), and acceleration ($a$) can each be graphed against time ($t$). Time is always on the horizontal ($x$) axis, with the other variable ($s$, $v$ or $a$) on the vertical ($y$) axis.

Because $s$, $v$ and $a$ are all related, a graph of any one of these variables against time can give information about the other variables:

- The gradient of an object's $s/t$ graph gives the object's $v$, for any particular value of $t$.
- The gradient of an object's $v/t$ graph gives the object's $a$, for any particular value of $t$.
- The area bound by an object's $v/t$ graph gives the object's $\Delta s$, for any particular $\Delta t$.
- The area bound by an object's $a/t$ graph gives the object's $\Delta v$, for any particular $\Delta t$.

Note that:

For an $s/t$ graph, $gradient = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{\Delta \vec{s}}{\Delta t} = \vec{v}$ . The formulas for gradient and velocity are identical.

Likewise:

For an $s/t$ graph, $gradient = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{\Delta \vec{v}}{\Delta t} = \vec{a}$ . The formulas for gradient and acceleration are identical.

Of course, if a $s/t$ or $v/t$ graph is curved, it is the gradient of the **tangent** to the curve that gives the $v$ or $a$ for any particular value of $t$.

Methods for finding the area bound by a graph within limits of time may be simple geometry, "square counting" or calculus.

## Constant Acceleration Formulæ

As long as the acceleration of an object is constant or "uniform" (doesn't change) the below formulæ relate all of the involved variables to allow for the solution of unknown magnitudes.

The derivation of these formulæ is from the combination and simplification of the mathematical procedures used to interpret the graphs described in the previous section.

Of course re-arrangement is often necessary for finding an unknown value. Also, an exam problem often requires more than one step to reach a final solution. For example, application of one formula may be required in order to gain the value of a variable needed to apply another formula so as to find a problem's required solution.

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{s} = \frac{\vec{u} + \vec{v}}{2} \times t$$

$$v^2 = u^2 + 2\vec{a}\vec{s}$$

Where $s$ = displacement, in m
$u$ = initial velocity, in $ms^{-1}$
$v$ = final velocity, in $ms^{-1}$
$t$ = time, in s
$a$ = acceleration, in $ms^{-2}$

**Danger** ☠: Again don't confuse the pronumeral ***s*** for displacement, with **s** for seconds, the unit of time.