Nicholas Hender

Area Of Study 1, Motion In One And Two Dimensions, Study Notes 3

Force

A force is a push or pull. A force can cause an object to do any of four things:

1. Start moving (accelerate; move with greater velocity).
2. Stop moving (accelerate negatively; move with less velocity; “decelerate”).
3. Change direction (accelerate at some angle to the direction in which it’s already moving).
4. Change shape (if enough force acts on an object that’s restricted from accelerating).

“Contact” forces push or pull things by touching them (such as the force applied to a cricket ball by a bat).

“Non-contact” forces push or pull things without touching them (such as the force applied by gravity to a falling object).

A measure of force is specified in units of **Newtons, N**.

Mass

Mass is a measure of how much of matter is contained within an object. The more matter from which an object is made, the more the mass of the object. The less matter from which an object is made, the less the mass of the object.

Danger ☠: Don’t confuse mass with weight. Mass and weight are different concepts. Weight is the downwards force of gravity acting on mass.

A measure of mass is specified in units of **kilograms, kg**.

Inertia

Inertia is the property of a stationary object to remain stationary. Inertia is why things stay still and don’t go anywhere, despite being untethered and free to move.

The more the mass of an object, the more its inertia, hence the term “inertial mass” (mass and inertial mass are synonymous). The more the inertia of an object, the more the force required to overcome its inertia and cause acceleration (commencement of motion).

More mass = more inertia = more force required to start moving. A bowling ball has more mass than a table-tennis ball. A bowling ball therefore has more inertia than a table-tennis ball. This is why a bowling ball needs more of a push than a table-tennis ball to start moving.

Momentum

Momentum is the property of a moving object to remain moving, in the same direction and at the same magnitude of velocity. Momentum involves an object’s velocity and therefore also its direction, so momentum is a **vector** quantity. Momentum is why moving things keep going and don’t accelerate, decelerate or change direction, as long as no force is applied.

Like inertia, an object’s momentum is affected by its **mass**. The more the mass of a moving object, the more its momentum.

The momentum of an object is also affected by its **velocity**. The more the velocity of a moving object, the more its momentum.

For example; a big truck, because of its mass, only needs a low magnitude of velocity to have a similar quantity of momentum to a small car with a relatively high magnitude of velocity. Also because of its mass, a big truck with a magnitude of velocity equal to that of a small car has a relatively much higher magnitude of momentum. It’s because of momentum that big trucks need powerful brakes, and that small cars don’t need power steering.

The more the momentum of an object, the more the force required to change its momentum and cause acceleration, deceleration or a change in direction. To quantify an object's momentum, **multiply its mass by its velocity**:

$$\vec{p} = m\vec{v}$$

Where m = mass, in kg
v = velocity, in ms^{-1}
p = momentum, in $kgms^{-1}$, or Ns (The value is the same, regardless of which unit is used. Ns is the preferred unit [because it's easier to write])

Impulse

If a stationary object is caused to move, or if a moving object is caused to accelerate, decelerate or change direction, the object has received an **impulse**. In each of these situations, the momentum of the object has changed, so one way to define impulse is as **a change in momentum**:

$$\vec{I} = \Delta\vec{p} = \vec{p}_2 - \vec{p}_1$$

Where Δp = change in momentum, in $kgms^{-1}$, or Ns
p_1 = initial momentum (before impulse), in $kgms^{-1}$, or Ns
p_2 = final momentum (after impulse), in $kgms^{-1}$, or Ns
I = Impulse, in $kgms^{-1}$, or Ns

Note that impulse, effectively being a quantity of momentum, has the same unit and is a vector quantity.

As already explained, it is **forces** acting on objects that cause stationary ones to move, and moving ones to accelerate, decelerate or change direction, so impulse is related to force. However, it's not only the quantity of force applied to an object that affects how much its momentum changes. The **time** for which the force is applied also has an affect.

- The more force applied to an object, the more the impulse, **and**
- The more time for which the force is applied, the more the impulse, **so**.....

Another way to define impulse is as a **quantity of force multiplied by a duration of time**:

$$\vec{I} = \vec{F}t$$

Where F = momentum, in N
t = time, in s
I = Impulse, in Ns, or $kgms^{-1}$

Note that it is this definition of impulse that gives the unit of Ns for momentum.

Because these two definitions of impulse are equivalent, they can be combined:

$$\vec{I} = \Delta\vec{p} = \vec{F}t$$

Considering this relationship between force and time, the impulse given to an object can also be found by calculating the area bound by a force/time graph within limits of time.

Impulse is demonstrated by accelerating or applying the brakes in a car. While accelerating or braking, constant force is applied, and momentum continues to change more and more for all of the time for which the force is applied.

Collisions

Collisions are when objects hit each other. Obviously when this happens the objects give each other impulses, so their momenta change. Momentum of objects when they collide is transferred and/or combined, depending on their relative velocities and the type of collision.

There are two types of collision; **elastic** and **inelastic**.

Basically (the following paragraph is an oversimplified generalisation), **Elastic collisions** are when the objects that collide "bounce" from each other, and remain as separate objects after the collision as they were before. **Inelastic collisions** are when the objects that collide "connect" with each other as the collision occurs, and are a single combined mass after the collision, rather than separate objects as they were before.

More precisely, a collision is ***elastic*** *if the total kinetic energy of all objects involved is conserved (equal before and after the collision), and* ***inelastic*** *if kinetic energy is lost (less after the collision than before).* The lost kinetic energy is transformed to heat, sound and light as the collision occurs. For an explanation of kinetic energy, see the following pages. Perfectly elastic collisions are almost only a theoretical concept, as in practice they are very rare, mainly due to friction.

In both elastic and inelastic collisions, **the total momentum of all objects *before* the collision is *equal* to the total momentum of all objects *after* the collision**. This is the **law of conservation of momentum**, as momentum is always conserved.

Quantitatively, for two objects **colliding elastically**:

$$\vec{p}_{total\_before} = \vec{p}_{total\_after}$$
$$\rightarrow \vec{p}_{1before} + \vec{p}_{2before} = \vec{p}_{1after} + \vec{p}_{2after}$$
$$\rightarrow m_1\vec{v}_{1before} + m_2\vec{v}_{2before} = m_1\vec{v}_{1after} + m_2\vec{v}_{2after}$$

Where p = momentum, in Ns, or kgms^{-1}
m = mass, in kg
v = velocity, in ms^{-1}

It can be assumed that the mass of each object is constant, so it's the velocities that change.

For two objects **colliding inelastically**:

$$\vec{p}_{total\_before} = \vec{p}_{total\_after}$$
$$\rightarrow \vec{p}_{1before} + \vec{p}_{2before} = \vec{p}_{after}$$
$$\rightarrow m_1\vec{v}_{1before} + m_2\vec{v}_{2before} = (m_1 + m_2)\vec{v}_{after}$$

Where p = momentum, in Ns, or kgms^{-1}
m = mass, in kg
v = velocity, in ms^{-1}

Again, it can be assumed that mass is not lost or gained, and that it's the velocity that changes. If objects collide, in most circumstances they were travelling in different directions before the collision, so it's most important to consider the directional components of the velocities when solving conservation of momentum problems.

Newton's Laws of Motion

In junior science it's probably adequate to merely memorise what these laws state. In senior physics it's paramount to understand what they mean and how they operate. They each refer to the concepts explained previously.

1. **For as long as there's no overall force acting on an object, the object will remain as it already is.**
   - A stationary object stays stationary, until a force causes it to start moving.
   - A moving object keeps moving, until a force causes it to accelerate, decelerate or change direction.
   - This law pertains directly to the concepts of **inertia** and **momentum**.

2. **The acceleration of an object is equal to the amount of force acting on it, divided by the object's mass.**

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\rightarrow \vec{F} = m\vec{a}$$

Where a = acceleration, in ms^{-2}
F = force, in N
m = mass, in kg

   - Commonly remembered as "force equals mass multiplied by acceleration".
   - Best understood as "more force = more acceleration" and "more mass = less acceleration", and vice versa: "less force = less acceleration" and "less mass = more acceleration".
   - A mathematical relationship between an object's mass and the force acting on it to quantify its acceleration.

3. **For every force on an object, there is another force, equal in magnitude and opposite in direction.**
   - Commonly remembered as "for every action there's a reaction".
   - This law justifies the concepts of inertia and momentum where forces appear to be unbalanced but aren't.
   - Demonstrated by:
     - A brick stationary on a table. The force of gravity is pulling the brick **downwards**, but it's not falling. Therefore the table **must** be pushing the book **upwards** with equal magnitude.
     - A light hanging by a chain from a ceiling. The force of gravity is pulling the light **downwards**, but it's not falling. Therefore the chain **must** be pulling the light **upwards** with equal magnitude.
     - A car cruising at constant velocity. The force of friction is pushing it **backwards**, but it's not decelerating. Therefore the car's engine **must** be pushing the car **forwards** with equal magnitude.

Energy and Work

Energy is the capacity to apply a force.
Basic examples:

- If you don't eat food, you'll have no energy, so won't have the capacity to apply the force required to come to school and learn, or to go to work and earn money.
- If you don't fill your car with petrol, it'll have no energy, so won't have the capacity to apply the force necessary to go anywhere.
- If you don't charge the batteries in your iPod, it'll have no energy, so won't have the capacity to apply the force necessary to spin its hard-disk or move the speakers in the earphones.

Energy can be categorised into many types. Machines, like car engines or electric power generators, operate by converting one form of energy into another that's more useful. The **law of conservation of energy states that energy cannot be destroyed or created, only converted from one form to another**. This means that the amount of energy that goes into a machine **must** be equal to the amount of energy that comes out. However, in reality the amount of **useful** energy that comes out of a machine is rarely anywhere close to the amount of energy that went in. This means that much of the energy that went in gets converted also into energy that's not useful and is wasted.

Work is defined as a **change in energy**. More specifically, it's a change in **one certain type** of energy. This allows work to be thought of as **an amount of energy converted** (or "consumed"). You feel tired after doing work because the energy you had has been used (converted into another form).

A measure of work or energy is a **scalar** quantity specified in units of **Joules, J**.

$$W = \Delta E = E_2 - E_1$$

Where W = work, in J
ΔE = change in energy, in J
E_1 = initial energy (before conversion), in J
E_2 = final energy (after conversion), in J

Work can also be defined as a magnitude of force applied for a certain distance:

$$W = Fd$$

Where W = work, in J
F = magnitude of force applied, in N
d = distance for which force has been applied, in m

Considering this relationship between force and distance, the amount of work done, or energy transferred, to an object can also be found by calculating the area bound by a force/distance graph within limits of distance.

Danger ☠: Don't confuse the pronumeral W for work, with **W** for Watts, the unit of power.

Kinetic Energy

"Kinetic" means "moving". If an object is moving, it has the capacity to apply a force. A moving object would apply a force in the event of a collision. Kinetic energy is the energy an object has because it's moving. It is the main kind of energy we are concerned with. Like momentum, kinetic energy is related to an object's mass and velocity, but in a different way. Unlike momentum, it's a scalar quantity.

$$E_K = \frac{1}{2}mv^2$$

Where E_K = kinetic energy, in J
m = mass, in kg
v = velocity, in ms^{-1}

Gravitational Potential Energy

Potential Energy can be thought of as "stored" of energy. The capacity to apply a force, but a capacity that's contained and waiting to be released. The simplest example of potential energy is a battery; it does nothing until connected to an electronic circuit, at which point the energy is released.

Gravitational potential energy is the energy an object has if gravity can make it move. Any object located above the ground has gravitational potential energy, because gravity can make it fall. When an object falls, **work** is done, as it's gravitational potential energy is converted to kinetic energy. Gravitational potential energy is related to three variables; mass, height, and acceleration due to gravity.

- The more the **mass** of an object, the more kinetic energy it will gain as it falls, so the more gravitational potential energy it has.
- The more the **height** of an object, the more kinetic energy it will gain as it falls, so the more gravitational potential energy it has.
- The more the **acceleration due to gravity** acting on an object, the more kinetic energy it will gain as it falls, so the more gravitational potential energy it has.

$$E_G = mgh$$

Where E_G = gravitational potential energy (of object), in J
m = mass (of object), in kg
g = acceleration due to gravity, in ms^{-2}
h = height (of object), in m

Not that ***g* is taken as a constant value of either 9.8 or 10** for situations in relative proximity to the Earth's surface.

Assuming that all E_G is converted to E_K as an object falls (which negates the force of friction due to air and is therefore a not a practical assumption), an object's E_G before falling is equal to its E_K the instant before its height reaches zero, so the formulæ for E_G and E_K are equivalent and can be combined. This helps in finding the value of unknown variables when solving problems involving falling objects.

$$E_G = E_K$$
$$\rightarrow mgh = \frac{1}{2}mv^2$$

Where E_G = gravitational potential energy, in J (of object **before** falling)
E_K = kinetic energy, in J (of object **after** falling, the instant before h=0)
m = mass (of object), in kg
g = 9.8 or 10 = acceleration due to gravity, in ms^{-2}
h = height, in m (of object before falling)
v = velocity, in ms^{-1} (of object after falling, the instant before h=0)

Hookes Law and Spring Potential Energy

A spring stores energy when stretched or compressed. As a stretched or compressed spring is released to return to its "natural" position, the stored energy is converted into kinetic energy. This is exactly what makes wind up toys and mechanical clocks work, and catapults, bear-traps, "pull-back" toy cars and rubber band powered model aeroplanes.

Hookes Law states that **the extension of a spring is directly proportional to the magnitude of the force that causes the extension**. The more force applied to stretching a spring, the more the spring stretches. A graph of the force acting on a spring against the spring's extension demonstrates a simple straight-line relationship. Force is usually shown on the vertical axis of a graph, and extension on the horizontal.

Some springs are "stiff" (like a suspension spring from a car), and some are "not-so stiff" (like a spring from a click-pen). As long as the relationship between the force applied to a spring and its extension is directly proportional, the "stiffness" of the spring, known as the **spring constant**, is given by the **gradient of a spring's force/extension graph**. A stiff spring will have a high spring constant, and a not-so stiff spring will have a low spring constant.

$$gradient = \frac{rise}{run} = \frac{\Delta F}{\Delta x} = \frac{F_2 - F_1}{x_2 - x_1} = k$$
$$\rightarrow F = kx$$

Where F = force (applied to spring), in N
x = extension (of spring), in m
k = spring constant, in Nm^{-1}

Note that:

- Hookes Law only applies to “ideal” (perfect) springs. Very few “real” (practical) springs demonstrate Hookes Law consistently. Hookes Law usually applies to within specific limits of extension or ranges of force. Many springs require a certain amount of force before starting to stretch and demonstrate Hookes Law.
- Extension is the length by which a spring is extended, not its complete length. Extension can therefore be calculated by subtracting a spring’s “natural” unstretched length from its total length when stretched by a certain magnitude of force.

$$x = total\_length\_of\_stretched\_spring - natural\_length\_of\_unstretched\_spring$$

The potential energy contained in an **ideal** spring with any particular constant is given by:

$$E_S = \frac{1}{2}kx^2$$

Where E_S = spring potential energy, in J
k = spring constant, in Nm^{-1}
x = extension (of spring), in m

Because most springs are not actually ideal, a **better method** for finding the potential energy of a stretched or compressed spring in practical situations is by finding the area bound by a force/extension graph within limits of extension.