Nicholas Hender

Area Of Study 1, Motion In One And Two Dimensions, Study Notes 4

Vertical Motion

Within the limits of our studies, **vertical motion** is motion in a straight line, and with constant (or “uniform”) acceleration, so if horizontal motion with constant acceleration has been properly understood, all that’s required to grasp this concept is to be aware that “vertical” is perpendicular to “horizontal”. Everything studied to this point in reference to horizontal motion (vector addition and subtraction, constant acceleration formulæ, Newton’s laws of motion etc.) are completely applicable to vertical motion as well.

The reason for which vertical motion is covered under a separate sub-heading is that some basic oversights are often made in the process of solving problems concerning vertical motion:

- Assuming there’s no air resistance and launch and landing heights are equal, for an object that moves up and then down, the magnitude of its velocity from the end of its initial impulse, at the start of its trajectory is maximum, and equal in magnitude to its velocity at the end of its trajectory. Hence, the initial velocity of the upwards stage of its motion is equal (in magnitude, but obviously opposite in direction) to the final velocity of the downwards stage. For example, a ball thrown directly upwards lands at the same speed at which it was launched, assuming it lands at the same height from which it was launched and that there’s no air friction.

- Also for an object that moves up and then down, the magnitude of its velocity at the peak of its trajectory is zero. Hence, the final velocity of the upwards stage of its motion is zero, and the initial velocity of the downwards stage of its motion is zero as well.

- Within proximity to the Earth’s surface (where nearly all situations concerning vertical motion are located), there’s **always a constant acceleration acting due to gravity**. The magnitude of this acceleration is considered as being **10ms^{-2}** (for simplicity), **9.8ms^{-2}** (for accuracy) or 9.82ms^{-2} (for being excessively pedantic). Which of these magnitudes to use for the correct solution of any specific problem depends on the textbook or exam paper in which the problem appears. In VCE examinations, always use whichever value is specified in the wording of the problem. While either 10ms^{-2} or 9.8ms^{-2} is acceptable, 10ms^{-2} is usually preferred.

- The constant acceleration due to gravity is always in a **downwards** direction. This seems obvious, but becomes especially important when it needs to be vectorially added to another acceleration in order to quantify the overall upwards or downwards acceleration of an object. For example, if a rocket is fired upwards, or a ball is thrown downwards. Usually the acceleration due to gravity is considered as **-**10ms^{-2} or **-**9.8ms^{-2}; the negative value indicating a downwards direction (because something launched in an upwards direction slows down as it rises). Sometimes, however, the acceleration due to gravity is considered as positive (because something dropped speeds up as it falls). The important thing about this is to remember to consider the direction of the acceleration due to gravity correctly **in relation to** any other accelerations experienced by an object in question. It will either be in the same direction, or in the opposite direction. Clearly this has a major influence over the solution to any problem concerning vertical motion.

- When only affected by the acceleration due to gravity, a vertically moving object’s velocity will:
  1. decrease as it rises, and
  2. increase as it falls, **so…**

- The vertical motion of an object is sometimes considered as **two separate stages**, or as one movement followed by another. In the example of a ball thrown directly upward:
  1. the stage during which the ball is moving upwards, and
  2. the stage during which the ball is moving downwards.

  In the example of a ball bounced of the ground:
  1. the stage during which the ball is approaching the ground, and
  2. the stage during which the ball is coming back up.

Nonetheless, to avoid concern over whether acceleration should be thought of as positive or negative, consider all downward vectors as negative values and all upward vectors as positive values; so acceleration is **always negative 10ms^{-2}**. This will enable the single application of constant acceleration linear motion formulæ for both upward and downward sections of motion, with a negative result indicating a downward direction.

Projectile Motion

When horizontal motion at constant acceleration is properly understood, and the details concerned with vertical motion at constant acceleration are remembered, a solid grasp of projectile motion should follow quite naturally.

Projectile motion can be generally defined as the motion of an object that travels **horizontally (along) simultaneously with travelling vertically** (up, down, or both one after the other, with the involvement of a gravitational acceleration). A **projectile** is any object that undergoes projectile motion. A **trajectory** is the path "traced out" by a projectile. **Range** is the horizontal displacement of a projectile (how far it goes relative to the ground. **Flight time** is the duration for which a projectile is "in the air".

Examples of projectiles and projectile motion:

- A ball rolling off the edge of a table. The ball keeps travelling horizontally as it falls vertically.
- A car driven off the edge of a cliff. The car keeps travelling horizontally as it falls vertically.
- A golf ball struck by a club. The ball travels horizontally as it rises and falls vertically.
- A cannonball fired from a cannon. The cannonball travels horizontally as it rises and falls vertically.

It's been demonstrated that the **vertical motion** of an object can be analysed in **two** main stages:

1. the **rising** stage, as a projectile moves upwards **slowing down** to a **final velocity of zero** at its peak, and
2. the **falling** stage, as a projectile moves downwards **speeding up** from an **initial velocity of zero** to reach the speed with which it was launched at the height from where it was launched.

Projectile motion can be analysed in **three** main stages:

1 & 2. as above for **vertical** motion, independent of a projectile's horizontal motion, considering only how it goes **up and/or down**, and

3. as the **horizontal** component of a projectile's motion, independent of its vertical motion, considering only how it goes **along**.

The only variable involved with a projectile's vertical motion that influences its horizontal motion is the time for which it is "in the air", or its "flight time". This affects the projectile's horizontal "range" (displacement), assuming that it stops moving when it lands.

Assuming there's no horizontal acceleration from the time of a projectile's launch (due to an impulse forwards or frictional forces backwards), **the horizontal component of a projectile's motion has constant velocity**, in accordance with Newton's first law of motion. Therefore, for such a projectile, **the only acceleration experienced is due to gravity**, which only affects the vertical component of its motion, causing speed to decrease upwards and increase downwards. These characteristics of projectile motion make trajectories **negatively parabolic**.

The **direction** of a projectile's motion at any point in its trajectory is **always** the resultant of the **vector addition of its horizontal and vertical velocities**. Often, information provided in a problem concerning projectile motion will include a single initial velocity and a launch angle to horizontal. This **initial velocity is a vector and must be resolved into its horizontal and vertical components** in order to ascertain the initial vertical velocity and the initial horizontal velocity. **Constant acceleration formulæ** can then be applied in order to calculate quantities such as flight time and range.

In practical situations, **air resistance** should be considered:

- **Horizontally**, air resistance is a **force of friction** (in the opposite direction to the motion), causing a **negative acceleration** as a projectile moves along (meaning that a projectile's horizontal velocity is **not** constant, but decreasing).
- **Vertically**, air resistance is also a frictional force, **adding to gravitational acceleration as a projectile rises**, and **subtracting from gravitational acceleration as a projectile falls** (this is why a parachutist lands slowly and hence safely).

Air resistance therefore affects **trajectory**:

- Vertical displacement (height) is reduced (for projectiles that travel upwards).
- Vertical (final) velocity is reduced (for projectiles that travel downwards).
- Horizontal (final) displacement is reduced.
- Horizontal velocity decreases with range (as displacement increases)

The trajectory of a projectile that experiences air resistance is henceforth **not** negatively parabolic. It resembles an negative parabola, but is slightly flatter and steeper on its fall; the gradient of the negative parabola's downward slope decreases (becomes more negative) more rapidly.

Exam problems concerning projectile motion will often involve different launching and landing heights (such as the trajectory of a projectile launched at an angle upwards from the top of a building), or a period for which the projectile experiences an impulse (such as a rocket powered projectile in which the rocket applies a constant force for an amount of time). It's necessary in these cases to break the motion of the projectile down into multiple stages, considering each stage of the motion separately, linking data from each stage of the trajectory to the next.