Nicholas Hender

Area Of Study 1, Motion In One And Two Dimensions, Study Notes 5

Circular Motion (in a Horizontal Plane); Period and Frequency

For an object travelling in a circle, the **period** is the time taken to travel once around the circle. The **frequency** is the number of times per second the object travels around the circle. Period is the inverse (or reciprocal) of frequency, so likewise, vice versa:

$$f = \frac{1}{T} = T^{-1}$$

$$T = \frac{1}{f} = f^{-1}$$

Where f = frequency, in Hz
T = period, in s

Circular Motion (in a Horizontal Plane); Speed

Speed is given by distance travelled divided by time taken (to travel that distance), so for an object travelling in a circle, speed is given by the circumference of the circle divided by the period.

For the circumference of a circle:

$$circumference = 2 \times \pi \times radius$$

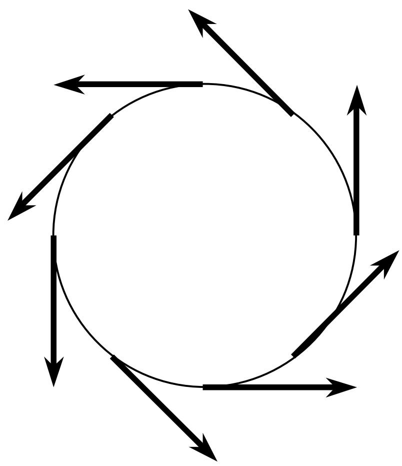
So, for the speed of an object travelling around the circle:

$$v = \frac{2\pi r}{T}$$

Where v = magnitude of velocity (speed), in ms^{-1}
r = radius (of circle), in m
T = period, in s

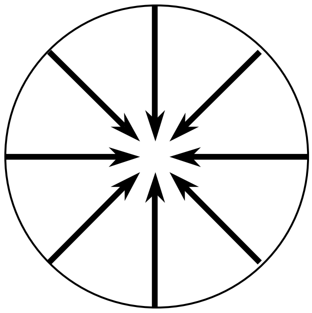
Circular Motion (in a Horizontal Plane); Velocity

Although the speed of an object travelling in a circle may be constant, its **velocity is always changing**, because while the velocity might have **constant magnitude**, the **direction of the velocity is tangential to the circle**. Velocity vectors for an object travelling in an anticlockwise circle:



Circular Motion (in a Horizontal Plane); Acceleration

If the direction of the velocity for an object travelling in a circle is always changing to be tangential to the circle, despite its magnitude remaining constant, there **must** be an acceleration. This acceleration is what causes the velocity's direction to be constantly changing. In order for the direction of the velocity to remain tangential to the circle, the direction of the velocity's **change** must be towards the centre of the circle, so the **acceleration is always towards the centre of the circle, at a right angle to the velocity**. Acceleration vectors for an object travelling in a circle:



To quantify the magnitude of the acceleration:

$$a = \frac{v^2}{r}$$

Where a = acceleration, in ms^{-2}
v = magnitude of velocity (speed), in ms^{-1}
r = radius (of circle), in m
T = period, in s

By combining this formula with that for the magnitude of velocity (previous page) and re-arranging:

$$a = \frac{v^2}{r} = \frac{2\pi v}{T} = \frac{4\pi^2 r}{T^2}$$

Where a = acceleration, in ms^{-2}
v = magnitude of velocity (speed), in ms^{-1}
r = radius (of circle), in m
T = period, in s

By substituting T with the formula for frequency (previous page) into this formula and simplifying:

$$a = \frac{v^2}{r} = \frac{2\pi v}{T} = \frac{4\pi^2 r}{T^2} = 4\pi^2 f^2 r$$

Where a = acceleration, in ms^{-2}
v = magnitude of velocity (speed), in ms^{-1}
r = radius (of circle), in m
T = period, in s
f = frequency, in Hz

Circular Motion (in a Horizontal Plane); Centripetal and Centrifugal Force

To provide the inwards acceleration necessary for circular motion, there needs to be an **inwards force**. This is **centripetal force**. Centripetal force can be quantified by combining Newton's second law of motion with the formulæ for acceleration (previous page):

$$F = ma = \frac{mv^2}{r} = \frac{m2\pi v}{T} = \frac{m4\pi^2 r}{T^2} = m4\pi^2 f^2 r$$

Where F = Force (centripetal), in N
m = mass (of object in circular motion), in kg
a = acceleration, in ms^{-2}
v = magnitude of velocity (speed), in ms^{-1}
r = radius (of circle), in m
T = period, in s
f = frequency, in Hz

Examples of centripetal force:

- The tension in a string (in the case of a mass being swung around in a circle on the end of a string).
- The friction between a car's tyres and a road (in the case of a car driving around a circular path).
- The contact between an outer ring and the object travelling circularly (in the cases of clothes being laundered during the spin cycle in a washing machine, people inside a "Gravitron" theme park ride, or a mixture being separated in a chemical "centrifuge").

If you've ever been in a "Gravitron" theme park ride, you'll have noticed that a strong force is experienced in a direction outwards from the circular path (making you "stick" to the inside wall, as though it was gravitational, hence the name "Gravitron", although it is **not** a gravitational force). This force is a **centrifugal force**, and is a reaction force to the centripetal force of the circular motion, in accordance with Newton's third law of motion (equal in magnitude but opposite in direction). As long as the radius of the circular path doesn't change, centrifugal and centripetal forces are equal in magnitude but opposite in direction. In circular motion, there needs to be a centrifugal force to restrain the centripetal force from decreasing the circular path's radius.

If an object in circular motion is released from the centripetal force, the acceleration ceases, and the object continues in a **straight line** with whatever velocity it had (in terms of magnitude **and** direction) at the point of being released. This is demonstrated by a ball bowled in a game of cricket and by a projectile launched from a catapult.