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Area Of Study 1, Motion In One And Two Dimensions, Study Notes 9

Gravity

Gravity is a force which attracts mass. Gravity is also generated by mass. The more the mass of an object, the more the force with which it will attract other mass. On the scale of what we can handle, regular objects do not have enough mass to show an easily measurable gravitational force. Only objects on an astronomical scale, such as stars and planets, have enough mass to obviously demonstrate gravitational force. It is the Earth's mass that provides the gravitational force we always experience. It attracts our mass allowing us to stay on its surface.

Gravitational force varies not only with mass, but also with distance. The more the distance between two masses, the less the force with which the masses are attracted. Gravitational force varies inversely with the square of the distance between two masses.

Gravitational Field Strength

This is a measure of the strength of an object's gravitational force, relative to its mass and distance. Gravitational force is always considered as originating from an object's centre of mass; the location within an object around which all of its mass is equally distributed, so distance is always measured from this point, and the direction of force is always towards this point.

$$g = \frac{Gm}{r^2}$$

Where g = gravitational field strength, in Nkg^{-1}
m = mass of object (**providing** the gravitational force), in kg
r = radius of separation (distance from object's centre of mass), in m
G = gravitational constant, **$6.67x10^{-11}$**

Gravitational field strength can be defined as an amount of force per unit of mass, or the number of Newtons of force experienced by a mass of 1kg, at any specified distance from another object's centre of mass, so:

$$g = \frac{Gm_1}{r^2} = \frac{F}{m_2} = a$$

Where g = gravitational field strength, in Nkg^{-1}
m_1 = mass of object (**providing** the gravitational force), in kg
r = radius of separation (distance from object's centre of mass), in m
G = gravitational constant, **$6.67x10^{-11}$**
F = force experienced by mass of 1kg, in N
m_2 = mass of object (**experiencing** the gravitational force), 1kg
a = g = acceleration due to gravitational force, in ms^{-2}

Note that the second half of this equation is Newton's second law of motion. Gravitational field strength can be specified with either Nkg^{-1} or ms^{-2} units, as they are both equal, but conventionally it's considered as Nkg^{-1}.

Newton's Law Of Universal Gravitation

Gravitational field strength only considers the force acting on a 1kg mass, and is generally applicable where one mass is particularly significant and others are negligible. Masses actually exert gravitational forces on each other. In situations such as our solar system, this matters because objects with comparable gravitational field strengths exist within proximity to each other.

Newton's law of universal gravitation gives the force of attraction between two masses with respect to the distance between them.

$F = m_1 a$.....................Newton's second law of motion

$a = g$..........................acceleration and gravitational field strength are equivalent

$F = m_1 g$.....................substituting g for a

$F = m_1 \times \frac{Gm_2}{r^2}$.......... substituting the formula for gravitational field strength for g

$F = \frac{Gm_1 m_2}{r^2}$............. simplifying

Where F = force of gravitational attraction between objects, in N
m_1 = mass of one object, in kg
m_2 = mass of another object, in kg
a = g = acceleration due to gravitational force, in ms^{-2}
g = gravitational field strength, in Nkg^{-1}
r = radius of separation (distance between the centre of mass of each object), in m
G = gravitational constant, **6.67×10^{-11}** Nm^2kg^{-2}

Re-arrangement of this formula shows the derived unit for the gravitational constant as Nm^2kg^{-2}.

Gravity And Circular Motion (Orbital Motion)

Newton's law of universal gravitation gives the force of attraction between two masses. Despite experiencing attractive force and being free to move, two masses in space do not necessarily bump into each other as a consequence. Instead, they can move **around** each other; the smaller mass **orbiting** the bigger mass.

The motion of a smaller mass around a bigger mass can be considered as **circular motion** (it's actually elliptical and the bigger mass moves slightly too). Because of this, formulæ associated with circular motion can also be applied to orbiting objects, and equalled and combined with formulæ associated with gravitation.

Gravitational force between objects provides (and is equal to) the centripetal force necessary for the acceleration keeping one mass in circular orbit around another. In keeping with the characteristics of circular motion, this acceleration is in the same direction as the force that causes it; always towards the centre of the circle. The direction of the velocity of the orbiting mass is always perpendicular to the acceleration, and hence tangential to the circle. The magnitude of the velocity (the speed) is constant. This is, of course, with the assumption (or approximation) that an orbiting object's motion is circular (and not elliptical, like it actually is).

$$v = \frac{2\pi r}{T} = \sqrt{\frac{Gm_1}{r}}$$

$a = g$ .... acceleration and gravitational field strength are equivalent, so:

$$\frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{Gm_1}{r^2}$$

$F_C = F_G$..acceleration and gravitational field strength are equivalent, so:

$$\frac{m_2 v^2}{r} = \frac{m_2 4\pi^2 r}{T^2} = \frac{Gm_1 m_2}{r^2}$$

Where v = velocity of orbiting object, in ms^{-1}
r = radius of separation (distance between the centre of mass of each object), in m
T = period of revolution, in s
m_1 = mass of central (larger) object, in kg
m_2 = mass of orbiting (smaller) object, in kg
a = g = acceleration due to gravitational or centripetal force, in ms^{-2}
F_C = centripetal force, in N
F_G = force of gravitational attraction between objects, in N
G = gravitational constant, $\mathbf{6.67 \times 10^{-11}} Nm^2kg^{-2}$

Kepler's Third Law

For every object orbiting a particular mass, the radius of separation cubed and divided by the period of revolution squared is equal. Therefore every mass has a constant value for all its orbiting objects.

$$\frac{r^3}{T^2} = \frac{Gm_1}{4\pi^2} = \text{constant}$$

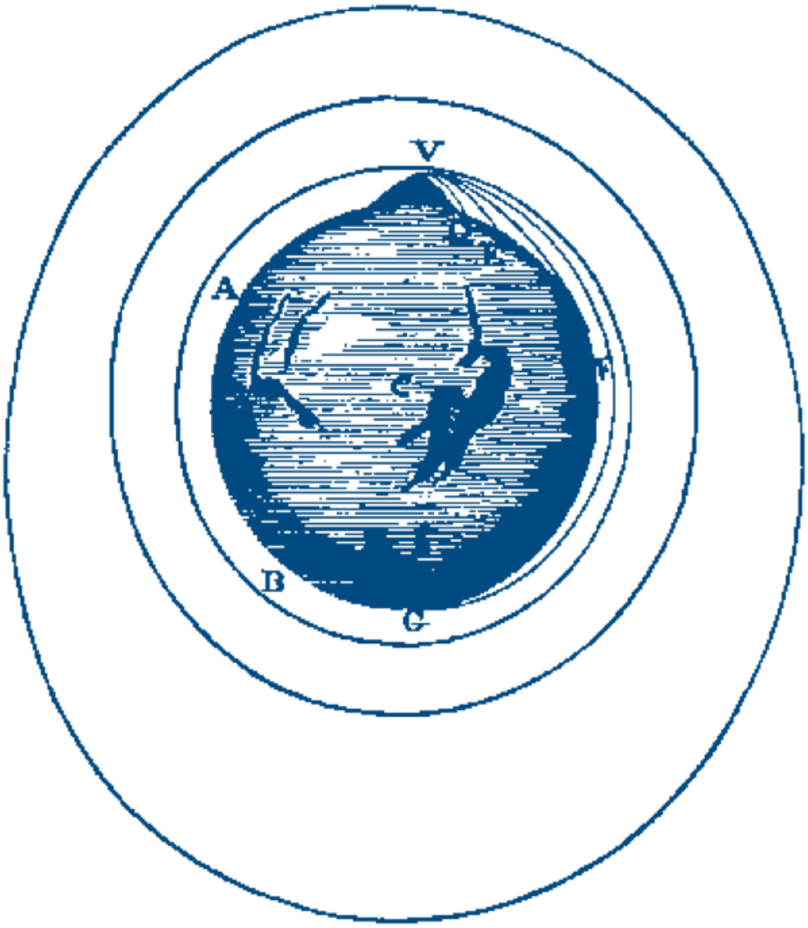
Where r = radius of separation, in m
T = period of revolution, in s
m_1 = mass of central (larger) object, in kg
G = gravitational constant, $\mathbf{6.67 \times 10^{-11}} Nm^2kg^{-2}$

Free-fall And Weightlessness

Weight is a force occurring due to gravity acting on a mass. While significantly less than on the surface of a planet, there is still gravitational force, and hence acceleration, in space, but objects in orbit appear to be floating rather than falling.

Orbiting objects are actually constantly falling, but have great velocity in a perpendicular direction (to their falling), such that for every tiny distance they fall, the more massive object at the centre of the orbit curves away by an equal tiny distance, so the two objects never get any closer.

Illustrating how this is possible, the below diagram, by Newton, shows the trajectories of a projectile fired at increasing velocities horizontal to the surface of the Earth.



Changes In Energy For Orbiting objects

An orbiting object has mass, height above a more massive object, and an acceleration due to gravity. Therefore it has gravitational potential energy, E_G, depending on these variables.

An orbiting object also has velocity, and therefore kinetic energy, E_K.

Applying force (probably with a small rocket) to change the orbital radius of such an object changes its height from the more massive object, and hence its E_G. Changing the orbital radius also changes its period of revolution, and therefore its velocity and E_K.

A **change in energy** is equivalent to an amount of **work done**, which is equal to the area bound within limits of a force/distance graph.

To find the work done in changing the radius of an object's orbit, and hence the object's change in energy, find the area bound within the limits of distance of its force/distance graph.