

Detailed Study, Materials And Their Use In Structures, Study Notes

Centre Of Mass

A **centre of mass** is the point within an object around which all its mass is equally distributed. More common but non-scientific terms for centre of mass are “centre of gravity” and “balance point”. Both these terms assume an object to exist in a gravitational environment, while an object’s centre of mass remains in the same place regardless of any action by gravity. However, in a gravitational environment an object will balance (not fall to one side or another) when suspended from its centre of mass. An object’s centre of mass is not necessarily at its geometric centre.

Unless specified otherwise, all forces associated with an object can generally be assumed to be acting towards or from its centre of mass. If specified otherwise, the forces associated with an object are acting at some distance from its centre of mass, or from where the object is attached to something else (by a hinge, for example). In these situations, the concept of torque must be considered (see below).

Levers

Levers are devices consisting of a beam (a “stick”) that is hinged at some point (called a **pivot** or a “fulcrum”) along its length. Two forces, called a **load** and an **effort** operate the lever.

There are three kinds of lever, arrived at by arranging the effort, load and pivot into their possible combinations of order:

- effort, pivot, load (example: a see-saw or a pair of scissors)
- pivot, effort, load (example: a fishing rod)
- effort, load, pivot (example: a door)

One lever may function with multiple loads and efforts, depending on the purpose of the lever.

Torque

Torque is force applied at some distance from either:

- an object’s centre of mass, or
- from where an object is attached (by a pivot) to something else.

$$\tau = Fr$$

Where F = Force, in N

r = radius of separation (distance between force and pivot), in m

τ = torque, in Nm

Levers being objects in which forces (“efforts” and “loads”) act at a distance from a pivot can be considered in terms of torque. All forces should be considered as acting around one single pivot (at a radius from only one point along the length of the lever).

Equilibrium

This relates directly to Newton's first and third laws of motion. If an object is stationary, it is in **equilibrium**, so all forces acting on it must amount to zero (the forces are "balanced"), and it will remain stationary until experiencing force greater in one direction than any others (the forces are unbalanced).

$$\Sigma F = 0, \text{ or}$$

$$\Sigma \tau = 0$$

$$= \tau_{\text{clockwise}} - \tau_{\text{anticlockwise}}$$

$$\rightarrow \tau_{\text{clockwise}} = \tau_{\text{anticlockwise}}$$

An object is considered as being in **stable equilibrium** when a **strong** force, relative to the forces holding the object stationary, is needed to make the object move.

An object is considered as being in **unstable equilibrium** when only a **weak** force, relative to the forces holding the object stationary, is needed to make the object move.

Bending

When studying motion, forces are considered as pushes or pulls on an object that cause it accelerate. If a force does not cause an object to accelerate it may instead cause it to **deform** (change shape, either temporarily or permanently).

Tension is when force (usually "pulling") is applied to an object in **outwards** directions, causing its length to increase (or "stretch").

Compression is when force (usually "pushing") is applied to an object in **inwards** directions, causing its length to decrease (or "shrink").

Shear is when force is applied to an object in a **sideways** direction (perpendicular relative to the object's length), causing the object to **bend**.

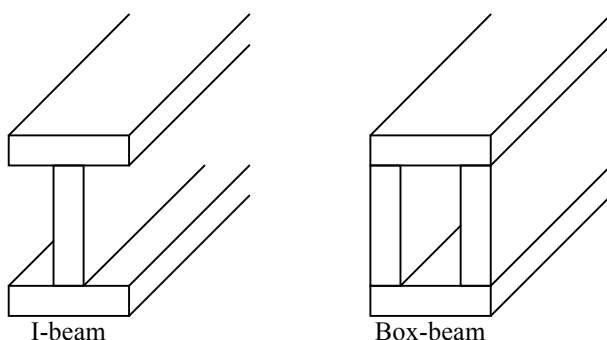
When shear causes an object to bend, the object undergoes tension and compression simultaneously:

- the side that bends **outwards** has increased length, so is in **tension**, and
- the side that bends **inwards** (the other side) has decreased length, so is in **compression**.

Beams

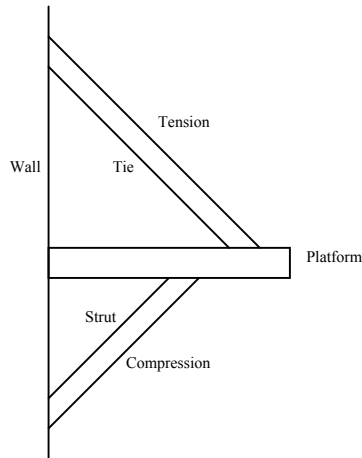
A beam (or "girder") is a device used in construction that can be considered as a lever (in terms of the torques acting on it around where its attached). A plank of timber is an example of a beam.

When shear is applied, **thick beams bend less and thin beams bend more**. Thick beams, however, have more mass, and their weight causes more compression in the columns that support them (**columns** can be considered as vertical and often cylindrical beams). They're also more expensive due to containing more material. Ideally, beams should be as light and resistant to bending as possible. The solution is "**I-beams**" and "**box-beams**". These are nearly as resistant to bending as solid beams, and have much less mass.



Platforms, or shelves, are attached to walls, and supported by beams called **struts** and **ties**.

- Struts attach from below a platform to a wall, making a triangle between the platform, the wall and the strut. Because they are below the weight they support, **struts are always in compression**.
- Ties attach from above of a platform to a wall, making a triangle between the platform, the wall and the tie. Because they are above the weight they support, **ties are always in tension**.



Arches

The components that are brought together to form an arch can be considered as short beams. Structures in the shape of an arch can support high loads because forces (such as weight) are “directed” into each component increasingly longitudinally; **downward forces perpendicular** to the length of short beams are gradually transferred into **compressive forces along** the length of other short beams. Shear forces have minimal bending effect because they act only on short beams, and hence result in very little torque. Generally, when under load, all of an arch is in compression.

Stress, Strength And Strain

Stress is an amount of force applied per unit of cross sectional area of a material. This is essentially the same as the scientific definition of **pressure**, but it is considered somewhat differently. Pressure is associated with forces applied to external surfaces, while stress is associated with forces acting internally, usually in terms of **tension** or **compression**.

$$\sigma = \frac{F}{A}$$

Where F = force (applied as tension or compression), in N
 A = cross sectional area (across which tension or compression is applied), in m^2
 σ = stress, in Nm^{-2} or Pascal, Pa

A material's **strength** is the amount of stress at which it **breaks** (fractures). Every material has its own specific value of strength.

Because the force that causes stress is applied "longitudinally" (along a material's length, as tension or compression), materials change length (deform) when in stress. The amount by which length changes compared to original length (without stress) is called **strain**.

$$\varepsilon = \frac{\Delta l}{l_1} = \frac{l_2 - l_1}{l_1}$$

Where Δl = change in length, in m
 l_1 = original length (without stress), in m
 l_2 = length with stress applied, in m
 ε = strain, (no unit; unit is cancelled by dividing one unit by another value of the same unit)

Young's Modulus

The amount of **stress** applied to a material can be graphed against the amount of **strain** caused by the stress, as a **stress/strain graph**. Such a graph demonstrates how a particular material will be strained when in stress. The gradient of a material's stress/strain graph is called the Young's modulus for that material.

$$\text{Young's modulus} = \text{gradient of stress/strain graph} = \frac{\sigma}{\varepsilon}$$

[for only the linear region of the stress/strain graph]

Where σ = stress, in Nm^{-2} or Pascal, Pa
 ε = strain

- The **higher** Young's modulus for a particular material, the **less** strain it will encounter when in stress.
- The **lower** Young's modulus for a particular material, the **more** strain it will encounter when in stress.

Examples:

- Young's modulus for **timber** is **high**. Timber **stretches little** (encounter **low strain**) when in **high tension**.
- Young's modulus for **rubber** is **low**. Rubber **stretches a lot** (encounters **high strain**) when in **low tension**.

Stiff And Flexible

A material with a relatively **high Young's modulus** experiences **low strain** under **high stress**. Such materials are considered to be **stiff**.

A material with a relatively **low Young's modulus** experiences **high strain** under **low stress**. Such materials are considered to be **flexible**.

Elastic And Plastic

For the linear region of a stress/strain graph, strain varies directly proportionally to stress. Work done in applying stress to the material is stored, and released kinetically when the stress is removed. In this way **materials act like springs and exhibit Hooke's law ($F=kx$)**.

Elastic behaviour of materials is such that they return to their original shape (or length), after a deforming force (of tension or compression) is removed. A material's elastic behaviour is demonstrated by the linear region of its stress/strain graph.

Plastic behaviour of materials is such that they remain deformed permanently (stretched or compressed) after a deforming force (of tension or compression) has been removed. A material's plastic behaviour is demonstrated by the non-linear region (after the linear region) of its stress/strain graph.

A material's **elastic limit** is the amount of stress at which it will no longer demonstrate completely **elastic behaviour** and begins to demonstrate **plastic behaviour**. It's the point on its stress/strain graph at which the **linear region** starts curving away to be **non-linear**.

Brittle And Ductile

Brittle materials break at, or not far beyond, their elastic limit. They demonstrate very little, if any, plastic behaviour, because they break before they deform such that they won't return to their original shape or length. Examples of particularly brittle materials are concrete and ceramic.

Ductile materials demonstrate extensive plastic behaviour before breaking. The plastic region of their stress/strain graphs show broad limits of strain that can be withstood before breakage, but without the ability to return to their original shape or length. Examples of particularly ductile materials are putty, clay and plasticine.

Strain Energy And Toughness

The **area bound within limits of strain** on a material's **stress/strain graph** gives the **strain energy per unit of volume** (in Jm^{-3}) for that material at the corresponding amount of stress.

This is equal to the **work done** (per unit of volume) in deforming (changing the length of) the material.

Within the elastic region of the material's stress/strain graph, this is equal to potential (spring) energy per unit of volume stored in the material.

The **Toughness** of a material is an **amount of strain energy per unit of volume** given by the area bound by a stress/strain graph at the point of breakage. That is, the area bound within the limit of strain at the material's **strength**. So a material's **toughness is the strain energy per unit of volume when the applied stress is equal to the material's strength**.

Composite materials

These are materials constructed from combinations. In composite materials, the stress/strain characteristics of one material compliment the stress strain characteristics of the other(s) with which it's combined. This results in materials that can be very strong and tough, while also having very little mass (and hence weight).

The most common example of a composite material is reinforced concrete. Concrete by itself has a low tensile strength and is brittle, but steel has a high tensile strength and can be quite ductile. By setting concrete around steel rods, the composite material (of concrete and steel) demonstrates the stress/strain characteristics of both concrete and steel. **In reinforced concrete beams, the steel rods are always placed near the surface of the concrete which will experience tension under load.** Reinforced concrete is also much lighter than an equivalent volume of only steel. Furthermore, steel and concrete have very similar rates of "thermal expansion" (they expand by the same amount as heat increases), so the combination is ideal.

Fibre glass and **carbon fibre** are other examples of composite materials. Glass and carbon are both strong but brittle, while the **resin** in which their fibres are set is ductile, tough and very light. The combination of resin with fibres of glass or carbon results in materials that are strong, tough and light, and suitable for use in things like tennis racquets, bicycle frames and yacht hulls.

Metal alloys are yet other common examples of composite materials. Elementary metals are mixed according to certain ratios in order to achieve materials with more suitable properties for particular applications. Brass is an alloy of copper and zinc, bronze is an alloy of copper and tin, and solder is an alloy of lead and tin. Stainless steel is an alloy of iron, chromium and nickel. The chromium and nickel are less susceptible to corrosion and prevent the iron from rusting.

Pre-stressed Materials

If the direction from which a material is to experience load is known before its construction, it can be manufactured to be **pre-stressed** in the opposite direction.

The most common example of this is flat truck trailers. A flat trailer will arch slightly **downwards** when loaded, such that the **top** is in **compression** and the **bottom** is in **tension**, and the entire structure is stressed. Pre-stressed trailers are welded together with the beams stressed such that they're arched slightly **upwards**. When not loaded, this makes the **top** of the trailer in **tension** and the **bottom** in **compression**. When it is loaded, the weight pushes it flat, so there's effectively no stress on the beams.

Another example of pre-stressing is embedding tensioned steel into wet concrete. When the concrete has set around the tensioned steel, the tension in the steel causes compression in the concrete. This allows the concrete to handle greater tensile force than otherwise, because the tensile forces (within limits) only stretch the concrete back to the length it would normally be.

Rivets that hold the iron sheeting together to make the hulls of large ships are also, in a way, pre-stressed. They are hammered closed when red hot. When they cool, they shrink, so the iron sheets they hold together are held together more tightly, but the rivets are permanently in tension.